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Teleportation of Any Form of Two-Mode Quantum States

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By using two two-mode Einstein–Podolsky–Rosen (EPR) pair eigenstates or two two-mode squeezed states as quantum channels we study the quantum teleportation of any form of two-mode quantum states, which conclude discrete and continuous variable quantum states.

KEY WORDS: teleportation; two-mode quantum states.

1. INTRODUCTION

Recently, the conception of entanglement becomes more and more fascinating and important as it play a central role for quantum communication, quantum teleportation, and quantum state engineering (Ekert and Josza, 1996; Divinvenzo, 1995; Bennett and Wisner, 1992; Bennett *et al*., 1993; Vaidman, 1994; Furasawa *et al*., 1998; Braunstein and Kimble, 1998). Two-particle, three-particle, and four-particle entanglement have been successfully demonstrated experimentally in trapped ions, Rydberg atoms, and cavity quantum electrodynamics (QED) (Turchette *et al*., 1998; Hagley *et al*., 1997; Rauschenbeutel *et al*., 2000; Sackett *et al*., 2000). In an entangled quantum state, measurements performed on one part of the system provide information on the remaining part, as first pointed out by Einstein, Podolsky, and Rosen (EPR) in their famous paper arguing the incompleteness of quantum mechanics (Einstein, Podolsky, and Rosen, 1935).

In Bennett and Wisner (1992), Bennett *et al*. (1993), and Vaidman (1994), Bennett *et al*. and Vaidman have both suggested that by virtue of entanglement it is possible to transfer the quantum state of a particle onto another particle provided that one does not obtain any information about the state in the course of this transformation. The experimental quantum teleportation was successfully performed by Zeilinger's groups (Bouwmeester, 1997) for a discrete variable system and Kimble's group for a continuous variable system. (Braunstein and Kimble, 1998; D'Ariano, Lo Presti, and Sacchi, 2000). The teleportation theory is further

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developed by D' Ariano, Presti, and Sacchi (2000) and Braunstein *et al.* (2000). Recently, by using the two-mode EPR pair eigenstates as quantum channels, Fan *et al.* studied the quantum teleportation of two-mode squeezed vacuum state (Hong-Yi and Yue, 2002). In this letter, we study the quantum teleportation of any form of two-mode quantum states (which conclude discrete and continuous variable quantum states) by using the two-mode EPR pair eigenstates or the two-mode squeezed vacuum as quantum channels.

2. EPR PAIR EIGENSTATES

The original conception of entanglement for bipartite is about wavefunction with continuous variables, as first pointed out by EPR in their famous paper arguing the incompleteness of quantum mechanics. (Einstein, Podolsky, and Rosen, 1935). EPR introduced the common eigenfunction of the relative position of two particles $\hat{X}_1 - \hat{X}_2$ (with their distance x_0) and their total momentum $\hat{P}_1 + \hat{P}_2$ as follow:

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \exp\left[ip(x_1 - x_2 + x_0)\right] dp \tag{1}$$

which describes a sharply correlated two-particle system. In Fan and Klauder (1994), Fan and Chen (1996), and Fan and Ye (1995), the common eigenstate $|\eta\rangle_{12}$ of commutative operators $(\hat{X}_1 - \hat{X}_2, \hat{P}_1 + \hat{P}_2)$ in two-mode Fock space (which we call the EPR pair eigenstate) is constructed

$$|\eta_{12}\rangle = \exp\left[-\frac{1}{2}|\eta|^2 + \eta a_1^+ - \eta^* a_2^+ + a_1^+ a_2^+\right]|00\rangle_{12} \tag{2}$$

where $\eta = (\eta_1 + i\eta_2)/\sqrt{2}$ is a complex number, $|00\rangle$ is the two-mode vacuum state, (a_i, a_i^+), $i = 1, 2$, are the two-mode Bose annihilation and creation operators in Fock space, related to $\hat{X}_i$ and $\hat{P}_i$ by

$$X_i = \frac{1}{\sqrt{2}}(a_i + a_i^+), \quad P_i = \frac{1}{\sqrt{2i}}(a_i - a_i^+) \tag{3}$$

It is easily proved that the state $|\eta\rangle_{12}$ obeys the eigenvector equations

$$\begin{aligned}(\hat{X}_1 - \hat{X}_2)|\eta\rangle_{12} &= \eta_1|\eta\rangle_{12}\\ (\hat{P}_1 - \hat{P}_2)|\eta\rangle_{12} &= \eta_2|\eta\rangle_{12}\end{aligned} \tag{4}$$

The state $|\eta\rangle_{12}$ is an entangled state, which can be seen from its Schmidt decomposition

$$|\eta\rangle_{12} = \exp\left(-\frac{i}{2}\eta_1\eta_2\right)\int_{-\infty}^{\infty} dx|x\rangle_1 \otimes |x - \eta_1\rangle_2 \exp(i\eta_2 x) \tag{5}$$

$$|\eta\rangle_{12} = \exp\left(-\frac{i}{2}\eta_1\eta_2\right)\int_{-\infty}^{\infty} dp|p+\eta_2\rangle_1 \otimes |-p\rangle_2 \exp(-ip\eta_1) \tag{6}$$

where $|x\rangle_i$ is the coordinate eigenstate of $\hat{X}_i$

$$|x\rangle_i = \pi^{-1/4}\exp\left[-\frac{1}{2}x^2 + \sqrt{2}xa_i^+ - \frac{1}{2}a_i^{+2}\right]|0\rangle_i \tag{7}$$

$|P\rangle_i$ is the momentum eigenvector of $\hat{P}_i$

$$|p\rangle_i = \pi^{-1/4}\exp\left[-\frac{1}{2}p^2 + i\sqrt{2}pa_i^+ + \frac{1}{2}a_i^{+2}\right]|0\rangle_i \tag{8}$$

Using the normal ordering form of the vacuum projection operator

$$|0\rangle\langle 0| =: e^{-a^+a} \tag{9}$$

where :: denotes the normal ordering, and the technique of integration within an ordered product (IWOP) of operator (Fan and Klauder, 1994; Fan and Chen, 1996; Fan and Ye, 1995) we can prove that $|\eta\rangle_{12}$ satisfies the completeness relation

$$\int \frac{d^2\eta}{\pi}|\eta\rangle_{12\,12}\langle\eta| = 1 \tag{10}$$

and the orthonormal property

$${}_{12}\langle\eta'|\eta\rangle_{12} = \pi\delta(\eta-\eta')\delta(\eta^*-\eta'^*) \tag{11}$$

Therefore, $|\eta\rangle_{12}$ makes up a new quantum mechanical representation. From Eq. (5) we operate the operator $\exp(i\hat{P}_1\hat{X}_2)$ on the state $|\eta\rangle_{12}$

$$\begin{aligned}
&\exp(i\hat{P}_1\hat{X}_2)|\eta\rangle_{12}\\
&\quad = \exp\left(-\frac{i}{2}\eta_1\eta_2\right)\int_{-\infty}^{\infty} dx\exp[i\hat{P}_1(x-\eta_1)]|x\rangle_1 \otimes |x-\eta_1\rangle_2 \exp(i\eta_2 x)\\
&\quad = \exp\left(-\frac{i}{2}\eta_1\eta_2\right)|x=\eta_1\rangle_1 \int_{-\infty}^{\infty} dy|y\rangle_2\exp(i\eta_2 y)\\
&\quad = \sqrt{2\pi}\exp\left(\frac{i}{2}\eta_1\eta_2\right)|x=\eta_1\rangle_1 \otimes |p=\eta_2\rangle_2
\end{aligned} \tag{12}$$

It then follows

$$\exp(-i\hat{P}_1\hat{X}_2)\sqrt{2\pi}\exp\left(\frac{i}{2}\eta_1\eta_2\right)|x=\eta_1\rangle_1 \otimes |p=\eta_2\rangle_2 = |\eta\rangle_{12} \tag{13}$$

We name $\exp(-i\hat{P}_1\hat{X}_2)$ the entangling operator since it entangles $|x=\eta_1\rangle_1$ (a coordinate eigenstate) and $|p=\eta_2\rangle_2$ (a momentum eigenstate) to the EPR pair eigenstate $|\eta\rangle_{12}$. Similarly, by operating the operator $\exp(i\hat{P}_2\hat{X}_1)$ on the state $|\eta\rangle_{12}$ we obtain

$$\exp(i\hat{P}_2\hat{X}_1)|\eta\rangle_{12} = \sqrt{2\pi}\exp\left(-\frac{i}{2}\eta_1\eta_2\right)|p=\eta_2\rangle_1 \otimes |p=\eta_1\rangle_{12} \quad (14)$$

$$\exp(-i\hat{P}_2\hat{X}_1)\sqrt{2\pi}\exp\left(-\frac{i}{2}\eta_1\eta_2\right)|p=\eta_2\rangle_1 \otimes |x=-\eta_1\rangle_2 = |\eta\rangle_{12} \quad (15)$$

Therefore, $\exp(-i\hat{P}_2\hat{X}_1)$ is another entangling operator.

3. TELEPORTATION OF ANY FORM OF TWO-MODE QUANTUM STATES

To teleport a two-mode quantum state, two quantum channels are necessary. Let particles 3 and 1 be prepared in an EPR pair eigenstate $|n\rangle_{13}$, and particles 4 and 2 be prepared in $|n'\rangle_{24}$. Allice and Bob share the two EPR pair eigenstates $|n\rangle_{13}$ and $|n'\rangle_{24}$ (two quantum channels). Initially the unknown two-mode quantum state $|\Psi\rangle_{56}$, which will be teleported from Allice to Bob, is in modes 5 and 6. Thus, the total initial state of the system is

$$|\Psi\rangle_{56} \otimes |\eta\rangle_{13} \otimes |\eta'\rangle_{24} \quad (16)$$

The teleportation scheme is as follows: A joint Bell measurement (quadrature phase measurement) of $\hat{X}_5 - \hat{X}_3$ and $\hat{P}_5 + \hat{P}_3$ performed on particles 5 and 3, and $\hat{X}_6 - \hat{X}_4$ and $\hat{P}_6 + \hat{P}_4$ on particles 6 and 4, respectively, projects the total initial state of the system onto the EPR entangled state $|\eta''\rangle_{53} \otimes |\eta'''\rangle_{64}$. The EPR pair eigenstate $|\eta''\rangle_{53} \otimes |\eta'''\rangle_{64}$ can be viewed as continous Bell basis, because they are orthogonal and complete. After the measurement, the projected state for particles 1 and 2 (the receiver Bob has particles 1 and 2) is

$${}_{64}\langle\eta'''| \otimes_{53} \langle\eta''|\Psi\rangle_{56} \otimes |\eta\rangle_{13} \otimes |\eta'\rangle_{24} \quad (17)$$

Substituting Eqs. (13) and (15) into Eq. (17), we have

$$\begin{aligned}
&{}_{64}\langle\eta'''| \otimes_{53} \langle\eta''|\Psi\rangle_{56} \otimes |\eta\rangle_{13} \otimes |\eta'\rangle_{24} \\
&= (2\pi)^2 \exp\left[\frac{i}{2}(\eta_1\eta_2 + \eta_1'\eta_2' + \eta_1''\eta_2'' + \eta_1'''\eta_2''')\right] \\
&{}_6\langle p=\eta_2'''| \otimes_4 \langle x=-\eta_1'''| \otimes_5 \langle p=\eta_2''| \otimes_3 \langle x=-\eta_1''| \\
&\exp(i\hat{P}_4\hat{X}_6)\exp(-i\hat{P}_2\hat{X}_4)\exp(i\hat{P}_3\hat{X}_5)\exp(-i\hat{P}_1\hat{X}_3) \\
&|x=\eta_1\rangle_1 \otimes |p=\eta_2\rangle_3 \otimes |x=\eta_1'\rangle_2 \otimes |p=\eta_2'\rangle_4 \otimes |\Psi\rangle_{56}
\end{aligned} \quad (18)$$

By means of the operator formula

$$\exp(\hat{A}+\hat{B}) = \exp(\hat{A})\exp(\hat{B})\exp(-\hat{C}/2) = \exp(\hat{B})\exp(\hat{A})\exp(\hat{C}/2) \quad (19)$$

where $[\hat{A}, \hat{B}] = \hat{C}$, $[\hat{C}, \hat{A}] = [\hat{C}, \hat{B}] = 0$, we have

$$\exp(i\hat{P}_4\hat{X}_6)\exp(-i\hat{P}_2\hat{X}_4) = \exp(-i\hat{P}_2\hat{X}_4)\exp(-i\hat{P}_2\hat{X}_6)\exp(i\hat{P}_4\hat{X}_6) \quad (20)$$

$$\exp(i\hat{P}_3\hat{X}_5)\exp(-i\hat{P}_1\hat{X}_3) = \exp(-i\hat{P}_1\hat{X}_3)\exp(-i\hat{P}_1\hat{X}_5)\exp(i\hat{P}_3\hat{X}_5) \quad (21)$$

Substituting Eqs. (20) and (21) into Eq. (18), we have

$$\begin{aligned}
&{}_{64}\langle\eta'''| \otimes_{53} \langle\eta''|\Psi\rangle_{56} \otimes |\eta\rangle_{13} \otimes \eta'\rangle_{24} \\
&= D_6\langle p = \eta_2'''| \otimes_4 \langle x = -\eta_1'''| \otimes_5 \langle p = \eta_2''| \otimes_3 \langle x = -\eta_1''| \\
&\exp(i\eta_1'''\hat{P}_2)\exp(-i\hat{P}_2\hat{X}_6)\exp(i\eta_2''\hat{X}_6)\exp(i\eta_1'''\hat{P}_1) \\
&\exp(-i\hat{P}_1\hat{X}_5)\exp(i\eta_2'''\hat{X}_5)|x = \eta_1\rangle_1 \otimes |P = \eta_2\rangle_3 \otimes |x = \eta_1'\rangle_2 \\
&\otimes|P = \eta_2'\rangle_4 \otimes |\Psi\rangle_{56} = 2\pi E_6\langle P = \eta_2'''| \otimes_5 \langle p = \eta_2''|\exp(i\eta_1'''\hat{P}_2) \\
&\exp(-i\hat{P}_2\hat{X}_6)\exp(i\eta_2'\hat{X}_6)\exp(i\eta_1''\hat{P}_1)\exp(-i\hat{P}_1\hat{X}_5)\exp(i\eta_2\hat{X}_5)|x = \eta_1\rangle_1 \\
&\otimes|x = \eta_1'\rangle_2 \otimes |\Psi\rangle_{56}
\end{aligned} \quad (22)$$

where we have used $\langle x|p\rangle = \frac{1}{\sqrt{2\pi}}\exp(ixp)$, and

$$D = (2\pi)^2 \exp\left[\frac{i}{2}(\eta_1\eta_2 + \eta_1'\eta_2' + \eta_1''\eta_2'' + \eta_1'''\eta_2''')\right] \quad (23)$$

$$E = \exp\left[\frac{i}{2}(\eta_1\eta_2 + \eta_1'\eta_2' + \eta_1''\eta_2'' + \eta_1'''\eta_2''' - 2\eta_1''\eta_2 - 2\eta_1'''\eta_2')\right] \quad (24)$$

Suppose that the teleported state $|\Psi\rangle_{56}$ is in any form of two-mode quantum state

$$|\Psi\rangle_{56} = \sum_{k,l} C_{kl}|k\rangle_5 \otimes |l\rangle_6 \quad (25)$$

By means of the completeness relation of coordinate eigenstates

$$\int_{-\infty}^{\infty} |q\rangle\langle q|dq = 1 \quad (26)$$

state $|\Psi\rangle_{56}$ can be written as

$$\begin{aligned}
|\Psi\rangle_{56} = \sum_{k,l} C_{kl}\frac{1}{\sqrt{2^{k+l}k!l!\pi}}\int_{-\infty}^{\infty} dq\, dq'|q\rangle_5 \otimes |q'\rangle_6 H_k(q)H_l(q') \\
\exp\left[-\frac{1}{2}(q^2 + q'^2)\right]
\end{aligned} \quad (27)$$

Substituting Eq. (27) into Eq. (22), we obtain

$${}_{64}\langle\eta'''| \otimes_{53} \langle\eta''|\Psi\rangle_{56} \otimes |\eta\rangle_{13} \otimes |\eta'\rangle_{24}$$

$$= 2\pi E_6\langle p = \eta_2'''| \otimes_5 \langle p = \eta_2''| \exp(i\eta_1'''\hat{P}_2)\exp(-\hat{P}\hat{X}_6)$$
$$\exp(i\eta_2'\hat{X}_6)\exp(i\eta_1''\hat{P}_1)\exp(-i\hat{P}_1\hat{X}_5)\exp(i\eta_2''\hat{X}_5)|x = \eta_1\rangle_1 \otimes |x$$
$$= \eta_1'\rangle_2 \otimes \sum_{k,l} C_{kl}\frac{1}{\sqrt{2^{k+l}k!l!\pi}}\int_{-\infty}^{\infty} dq\, dq'|q\rangle_5 \otimes |q'\rangle_6 H_k(q)H_l(Q')$$
$$\exp\left[-\frac{1}{2}(q^2 + q'^2)\right] = E\sum_{k,l} C_{kl}\frac{1}{\sqrt{2^{k+l}k!l!\pi}}\int_{-\infty}^{\infty} dq\, dq' H_k(q)H_l(q')$$
$$\exp\left[-\frac{1}{2}(q^2 + q'^2)\right]\exp[i(\eta_2' - \eta_2''')q' + i(\eta_2 - \eta_2'')q]|x = \eta_1 - \eta_1'' + q\rangle_1$$
$$\otimes|x = \eta_1' - \eta_1''' + q'\rangle_2 = EU\sum_{k,l} C_{kl}\frac{1}{\sqrt{2^{k+l}k!l!\pi}}\int_{-\infty}^{\infty} dq\, dq' H_k(q)H_l(q')$$
$$\exp\left[-\frac{1}{2}(q^2 + q'^2)\right]|q\rangle_1 \otimes |q'\rangle_2 \tag{28}$$

where we have used

$$\exp(-i\hat{P}y)|x\rangle = |x + y\rangle \tag{29}$$
$$U = \exp\lfloor -i\hat{P}_1(\eta_1 - \eta_1'') - i\hat{P}_2(\eta_1 - \eta_1''')\rfloor$$
$$\exp\lfloor i(\eta_2 - \eta_2'')\hat{X}_1 + i(\eta_2' - \eta_2''')\hat{X}_2\rfloor \tag{30}$$

Comparing Eqs. (27) and (28) we can see that up to a simple unitary transformation U and a phase factor E, the outcome states in modes 1 and 2 are the same as the incoming unknown two-mode quantum states in modes 5 and 6. If Alice sends the results of the measurement $(\hat{X}_5 - \hat{X}_3, \hat{P}_5 + \hat{P}_3)$ and $(\hat{X}_6 - \hat{X}_4, \hat{P}_6 + \hat{P}_4)$ to the receiver Bob (classical information delivery), after making a unitary transformation to erase the phase factor and a unitary transformation U^{-1}, he can obtain the unknown two-mode quantum state. Thus the teleportation of any form of two-mode quantum states given by Eq. (25) is carried out.

In the above discussion, we use two-mode EPR pair eigenstates as quantum channels to study the quantum teleportation of any form of n-mode quantum states. For a more practical quantum channel, two-mode squeezed vacuum state, which can be realized experimentally, is a good candidate. We now proceed to study the quantum teleportation of any form of n-mode quantum states through n two-mode squeezed vacuum state channels. Suppose that the quantum channels which Alice and Bob share are n two-mode squeezed vacuum states $|\tau_i\rangle_{n+i,2n+i} = S_{n+i,2n+i}(\tau_i)|00\rangle_{n+i,2n+i}$, $(i = 1, 2, \ldots, n)$, where $S_{i,j}$ is two-mode squeezed operator for modes i and j, and Alice initially possesses an unknown quantum state $|\Psi\rangle_{1,2,\ldots,n}$, which will be teleported from Alice to Bob. Thus the total initial state of the system is $|\Psi\rangle_{1,2,\ldots,n} \otimes |\tau_1\rangle_{n+1,2n+1} \otimes |\tau_2\rangle_{n+2,2n+2} \otimes \cdots \otimes$

$|\tau_n\rangle_{2n,3n}$. To teleport the target state $|\Psi\rangle_{1,2,\ldots,n}$, Alice makes a joint measurement on modes $(n+i, 2n+i)$, $(i = 1, 2, \ldots, n)$, which leads to another n squeezed vaccum states, say $|\tau_i\rangle_{n+i,2n+i} = S_{n+i,2n+i}(\tau_i)|00\rangle_{n+i,2n+i}$, $(i = 1, 2, \ldots, n)$, $|\tau'\rangle_{35} = S_{35}(\tau')|00\rangle_{35}$, and $|\sigma\rangle_{46} = S_{46}(\sigma')|00\rangle_{46}$. By means of the expression of squeezed operator in the two-mode EPR entangled state representation (Hong-Yi, 1997)

$$S_{ij}(\mu) = \exp\left[\lambda(a_i^+ a_j^+ - a_i a_j)\right] = \frac{1}{\mu\pi}\int d^2\eta|\eta/\mu\rangle_{ij\ ij}\langle\eta|, \quad \mu = e^{\lambda} \tag{31}$$

and Eq. (5), we can obtain the projected state for modes 1 and 2

$$\begin{aligned}
&{}_{35}\langle\tau'| \otimes_{46} \langle\sigma'|\tau\rangle_{13} \otimes |\sigma\rangle_{24} \otimes |\psi\rangle_{56} \\
&= {}_{35}\langle 00| \otimes_{46} \langle 00|S_{35}^+(\tau')\, S_{46}^+(\sigma')\, S_{13}(\tau) S_{24}(\sigma)|00\rangle_{13} \otimes |00\rangle_{24} \otimes |\psi\rangle_3 \\
&= \frac{1}{\tau\tau'\sigma\sigma'\pi^4}\int d^2\eta\, d^2\eta'\, d^2\eta''\, d^2\eta'''\, \exp\left[-\frac{1}{2}(|\eta|^2 + |\eta'|^2 + |\eta''|^2 + |\eta'''|^2)\right] \\
&{}_{35}\langle\eta'/\tau'| \otimes_{46} \langle\eta''/\sigma'|\eta'''/\tau\rangle_{13} \otimes |\eta/\sigma\rangle_{24} \otimes |\psi\rangle_{56} \\
&= \frac{1}{\tau\tau'\sigma\sigma'\pi^4}\int d^2\eta\, d^2\eta'\, d^2\eta''\, d^2\eta'''\, \exp\left[-\frac{1}{2}(|\eta|^2 + |\eta'|^2 + |\eta''|^2 + |\eta'''|^2)\right] \\
&\exp\left[\frac{i}{2}\left(\frac{\eta_1'\eta_2'}{\tau'^2} + \frac{\eta_1''\eta_2''}{\sigma'^2} + \frac{\eta_1'''\eta_2'''}{\tau^2} + \frac{\eta_1\eta_2}{\sigma^2}\right)\right] \\
&\int_{-\infty}^{\infty} dx_1\, dx_{25}\langle x_1 - \eta_1'/\tau'| \otimes_6 \langle x_2 - \eta_1''/\sigma'|\psi\rangle_{56} \otimes \big|x_1 + \eta_1'''/\tau\rangle_1 \otimes | \\
&x_2 + \eta_1/\sigma\rangle_2 \exp\left[-i\left(\frac{\eta_2'}{\tau'} - \frac{\eta_2'''}{\tau}\right)x_1 - i\left(\frac{\eta_2''}{\sigma'} - \frac{\eta_2}{\sigma}\right)x_2\right]
\end{aligned} \tag{32}$$

Substituting Eq. (27) into Eq. (32), we have

$$\begin{aligned}
&{}_{35}\langle\tau'| \otimes_{46} \langle\sigma'|\tau\rangle_{13} \otimes |\sigma\rangle_{24} \otimes |\psi\rangle_{56} \\
&= \frac{1}{\tau\tau'\sigma\sigma'\pi^4}\int d^2\eta\, d^2\eta'\, d^2\eta''\, d^2\eta'''\, \exp\left[-\frac{1}{2}(|\eta|^2 + |\eta'|^2 + |\eta''|^2 + |\eta'''|^2)\right] \\
&\exp\left[\frac{i}{2}\left(-\frac{\eta_1'\eta_2'}{\tau'^2} - \frac{\eta_1''\eta_2''}{\sigma'^2} + \frac{\eta_1'''\eta_2'''}{\tau^2} + \frac{\eta_1\eta_2}{\sigma^2} + \frac{2\eta_1'\eta_2'''}{\tau\tau'} + \frac{2\eta_1'\eta_2'''}{\sigma\sigma'}\right)\right] \\
&\sum_{k,l} C_{kl}\frac{1}{\sqrt{2^{k+l}k!l!\pi}}\int_{-\infty}^{\infty} dq\, dq' H_k(q) H_l(q')\, \exp\left[-\frac{1}{2}(q^2 + q'^2)\right] \\
&|q + \eta_1'/\tau' + \eta_1'''/\tau\rangle_1 \otimes |q' + \eta_1''/\sigma' + \eta_1/\sigma\rangle_2 \\
&\exp\left[-i\left(\frac{\eta_2'}{\tau'} - \frac{\eta_2'''}{\tau}\right)q - i\left(\frac{\eta_2''}{\sigma'} - \frac{\eta_2}{\sigma}\right)q'\right]
\end{aligned}$$

$$= \hat{F}(\hat{X}_1, \hat{X}_2, \hat{P}_1, \hat{P}_2) \sum_{k,l} C_{kl} \frac{1}{\sqrt{2^{k+l}k!l!\pi}} \int_{-\infty}^{\infty} dq\, dq'\, H_k(q) H_l(q')$$

$$|q\rangle_1 \otimes |q'\rangle_2 \exp\left[-\frac{1}{2}(q^2 + q'^2)\right] \tag{33}$$

Where

$$\hat{F}(\hat{X}_1, \hat{X}_2, \hat{P}_1, \hat{P}_2)$$

$$= \frac{1}{\tau\tau'\sigma\sigma'\pi^4} \int d^2\eta\, d^2\eta'\, d^2\eta''\, d^2\eta''' \exp\left[-\frac{1}{2}(|\eta|^2 + |\eta'|^2 + |\eta''|^2 + |\eta'''|^2)\right]$$

$$\exp\left[\frac{i}{2}\left(-\frac{\eta_1'\eta_2'}{\tau'^2} - \frac{\eta_1''\eta_2''}{\sigma'^2} + \frac{\eta_1'''\eta_2'''}{\tau^2} + \frac{\eta_1\eta_2}{\sigma^2} + \frac{2\eta_1'\eta_2'''}{\tau\tau'} + \frac{2\eta_1''\eta_2}{\sigma\sigma'}\right)\right]$$

$$\exp\left[-i\left(\frac{\eta_1'}{\tau'} + \frac{\eta_2'''}{\tau}\right)\hat{P}_1 - i\left(\frac{\eta_1''}{\sigma'} + \frac{\eta_1}{\sigma}\right)\hat{P}_2\right]$$

$$\exp\left[-i\left(\frac{\eta_2'}{\tau'} - \frac{\eta_2'''}{\tau}\right)\hat{X}_1 - i\left(\frac{\eta_2''}{\sigma'} - \frac{\eta_2}{\sigma}\right)\hat{X}_2\right] \tag{34}$$

Alice tells her measurement result to Bob through a classical channel, Bob then perform a transformation of $[\hat{F}^{-1}(\hat{X}_1, \hat{X}_2, \hat{P}_1, \hat{P}_2)$ to obtain the state $|\psi\rangle_{12}$ imitating the state $|\psi\rangle_{56}$. Thus the teleportation of any form of two-mode quantum states through a two-mode squeezed state channel is carried out.

4. SUMMARY

In summary, by means of the entagling operators for the two-mode EPR pair eigenstate, we study the quantum teleportation of any form of two-mode quantum states (which conclude discrete and continuous variable quantum states) through two EPR pair eigenstate channels. With the help of the expression of two-mode squeezed vacuum state in the two-mode EPR pair eigenstate representation, we derive the results of teleporting any form of two-mode quantum states through two two-mode squeezed vacuum state channels. Our calculation has been greatly simplified by virtue of the EPR pair eigenstates $|\eta\rangle$. So far as we know, in the literature about quantum teleportation the advantages of using two-mode EPR pair eigenstate have not been paid enough attention.

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